Time dependence of c in FLRW metrics

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Can we be certain that the velocity of c must be constant over all history? If it is not, how could we interpret the situation? Let us see what happens if we assume that the velocity of light is a function of some kind. It can not depend on space-coordinates because we expect that the laws of physics do not depend on the position in space. Still it might depend on time.

The standard form of the FLRW line-element:

$$c^2\ d\tau^2 = c^2\ dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2\ d\theta^2 + r^2\ \sin(\theta)^2\ d\phi^2 \right) \tag{1}$$

Consider now a slightly modified FLRW line-element, where we in stead of the constant c have a time dependent "velocity of light" $C(t)$. The scale function $a(t)$ then becomes an other we denote $\mathcal{A}(t)$. Equation (1) the gets the form:

$$C(t)^2\ d\tau^2 = C(t)^2\ dt^2 - \mathcal{A}(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2\ d\theta^2 + r^2\ \sin(\theta)^2\ d\phi^2 \right) \tag{2}$$

We can now define a new function $\mathcal{B}(t)$ such as:

$$C(t) = c\ \mathcal{B}(t) \tag{3}$$

where c expresses the value of the velocity of light, as it is measured to day. Then by division of both sides of equation (2) with $\mathcal{B}(t)$ we get:

$$c^2\ d\tau^2 = c^2\ dt^2 - \left(\frac{\mathcal{A}(t)}{\mathcal{B}(t)} \right)^2 \left(\frac{dr^2}{1 - kr^2} + r^2\ d\theta^2 + r^2\ \sin(\theta)^2\ d\phi^2 \right) \tag{4}$$

Further denote

$$a(t) = \frac{\mathcal{A}(t)}{\mathcal{B}(t)} \tag{5}$$

and it becomes:

$$c^2\ d\tau^2 = c^2\ dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2\ d\theta^2 + r^2\ \sin(\theta)^2\ d\phi^2 \right) \tag{1*}$$

We see that we get back the original FLRW line-element (1) . Thus we can conclude that it is the most general one. Everything that can vary can be included in the scale factor $a(t)$. Solving it gives us what there is to know of this particular geometric model of the universe.